ELASTIC PROPERTIES OF ANISOTROPIC ACOUSTO-OPTIC CRYSTALS TELLURIUM DIOXIDE AND CALOMEL

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ABSTRACT

The goal of the research consists in the investigation of light diffraction by ultrasound propagating in crystalline media. The major attention during the investigation was devoted to the conditions of acoustic propagation and reflection in the crystals possessing strong anisotropy of their elastic properties. Single crystals of paratellurite TeO_2 and calomel Hg_2Cl_2 have been chosen for the analysis due to the large obliquity angles between phase and group velocities of ultrasound in the materials. The paper also discusses possible applications of the examined phenomena in Acousto-Optics.

Keywords: acoustic phase and group velocity, obliquity angle, paratellurite, calomel, tunable acousto-optic filters

1. INTRODUCTION

The paper describes some results of investigation of light diffraction by ultrasound propagating in crystalline media. The major attention during the investigation was devoted to conditions of acoustic propagation and reflection in crystals possessing strong anisotropy of their elastic properties – Ref. 1-5. The grade of the elastic anisotropy may be qualitatively and quantitatively evaluated if the dependence is known of sound velocity on directions of acoustic propagation in a crystal - Ref. 1. Among other materials possessing the strong dependence of the phase velocity on the direction of sound propagation, the single crystals of paratellurite TeO_2 and calomel Hg_2Cl_2 have been chosen for the analysis during the present research - Ref. 4-5.

2. CALCULATION OF ACOUSTIC PHASE VELOCITIES

There exists a relatively simple method to calculate the values of the acoustic phase velocities V in the crystalline materials. The method is based on application of the Christoffel equation valid both for the isotropic and in the elastically anisotropic media - Ref. 1-2. The Christoffel equation may be written as follows:

$$\Gamma_{ijkl} u_l = \rho V^2 u_i \quad , \qquad (1)$$

where $\Gamma_{ijkl} = c_{ijkl} n_j n_k$ - component of Christoffel tensor, u_i -acoustic polarization, ρ - density, V - phase velocity, c_{ijkl} - elastic coefficients, n_i - unit vectors orthogonal to acoustic wave front.

As a result, it is possible to obtain the equations to determine the intrinsic vectors u_i and the corresponding values of the phase velocity V in a crystal. It is known that the crystals of paratellurite and calomel belong to the tetragonal structure. Substituting corresponding values of the elastic constants and solving the equation, in a general case, it is possible to show that there exist three plane linearly polarized elastic waves with mutually orthogonal polarizations corresponding to the given direction of sound propagation n_i .

3. ACOUSTIC PHASE VELOCITIES IN PARATELLURITE AND CALOMEL

Solving the Christoffel equation for the case of the tetragonal crystals, it is possible to derive the following expressions for the values of the acoustic phase velocities V in the XY plane of the materials – Ref. 1:

$$V_1^2 = \frac{c_{44}}{\rho}, \qquad (2)$$

$$V_2^2 = \left(\frac{1}{2\rho}\right)\{c_{11} + c_{66} + [(c_{11} - c_{66})^2 \cdot \cos^2 2\varphi + (c_{12} + c_{66})^2 \cdot \sin^2 2\varphi]^{0.5}\} \qquad (3)$$

$$V_3^2 = \left(\frac{1}{2\rho}\right)\{c_{11} + c_{66} - [(c_{11} - c_{66})^2 \cdot \cos^2 2\varphi + (c_{12} + c_{66})^2 \cdot \sin^2 2\varphi]^{0.5}\} \qquad (4)$$

The phase velocities and the corresponding intrinsic vectors were found for each of the directions in XY plane of the tetragonal crystals. Data obtained as a result of the carried out calculation on base of Eq.(2)– (4) are plotted in Fig. 1 and Fig. 2. They illustrate the slowness curves *1/V* in the materials. The figures show only two of the three slowness curves: the quasi-longitudinal QL acoustic mode and the quasi-shear QS wave. As known, the second shear mode in XY plane is isotropic and it was not included in the picture – Ref. 1.

If one compares the slowness curves of paratellurite TeO_2 and calomel Hg_2Cl_2 he finds that the slowness curves for tellurium

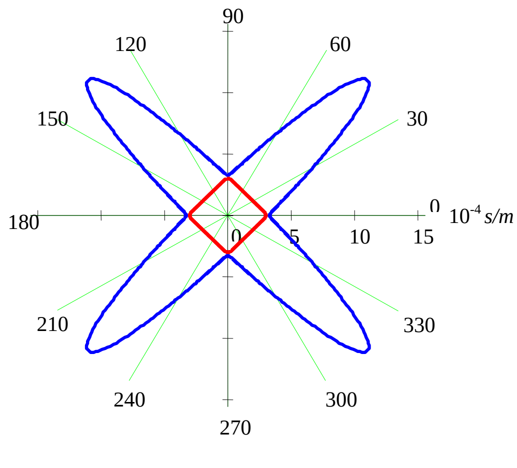

Figure 1. Slowness surface in XY plane of paratellurite

dioxide are included inside the curves corresponding to calomel. It means that the single crystal of calomel possesses lower acoustic phase velocity than paratellurite. The calculations also showed that the ratio of the maximum and the minimum phase velocity values in XY plane of paratellurite is equal to 5. It was calculated that the corresponding ratio of the phase velocities in calomel is slightly lower, e.g. 3.6.

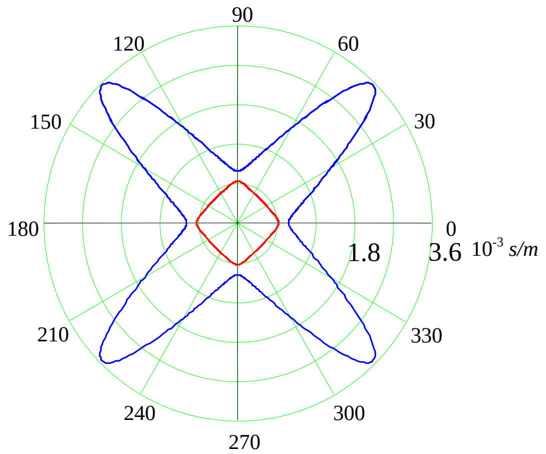

Figure 2. Slowness surface in XY plane of calomel

4. OBLIQUITY ANGLE OF ACOUSTIC ENERGY

In order to determine the direction of the energy flow in a crystal, it is necessary to find the acoustic obliquity angle, i.e. walk-off angle Ψ - Ref. 1. The walk-off angle is defined as the angle between the phase and the group velocities of ultrasound. As known, the group velocity coinciding with the acoustic energy flow is directed along a normal to the surface of slowness. The value of the acoustic walk-off angle may be given by the expression:

$$\varphi = \arccos\left(\frac{1}{V_p} \cdot \frac{dV_p}{d\varphi} - \frac{\pi}{2}\right) \qquad (5)$$

The present research proved that there are directions in paratellurite and calomel single crystals along which the walk-off angle exceeds $\Psi > 50^0$. For example, the maximum value of the walk-off angle in TeO_2 is equal to $\Psi = 74^0$ if the acoustic wave is sent in the crystal at the angle $\alpha = 38^0$

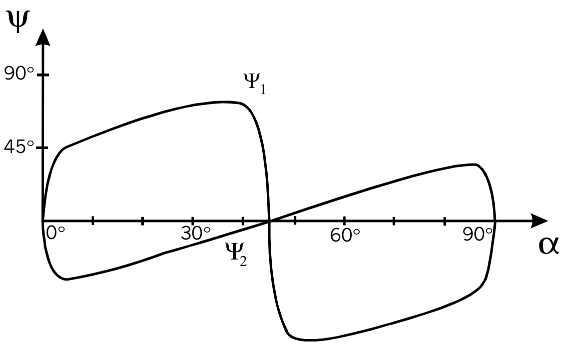

Figure. 3. Acoustic obliquity angles in paratellurite

relatively to X axis in XY plane of the material. The obliquity angle in calomel is also very large $\Psi = 70^0$. This regular trend illustrate data plotted in Fig. 4. The analysis also proves that at $\alpha = 0$, $\alpha = 45^0$ and $\alpha = 90^0$, the acoustic energy walk-off is absent in the case of the tetragonal crystals.

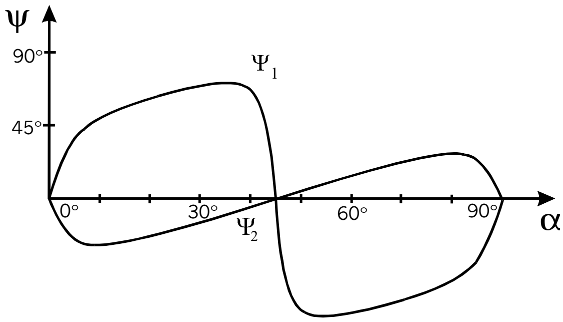

Figure 4. Acoustic obliquity angles in calomel

5. REFLECTION OF ACOUSTIC WAVES AT FREE BOUNDARY IN CRYSTALS

A peculiar reflection of the acoustic waves from a free boundary in the anisotropic crystals was also examined in the paper. This reflection is shown in Fig. 5. As illustrated, a wave reflection from the boundary is accompanied by the condition of equal tangential components of the wave vectors of an incident and a reflected acoustic beam – Ref. 1.

Graphic illustration of the mentioned law related to the tangential wave vector components is presented in Fig. 6. The analysis proves that one can fiond a refelction in a crystal when the elastic energy of a reflected acoustic wave is directed approximately backwards, i.e. towards the energy of the incident acoustic wave - Ref. 2. If the angle Ω is defined as the angle between the energy flows of the incident and the reflected acoustic waves then it may be also defined as the angle of separation of the two acoustic beams in space. This statement is illustrated in Fig. 5 and Fig. 6.

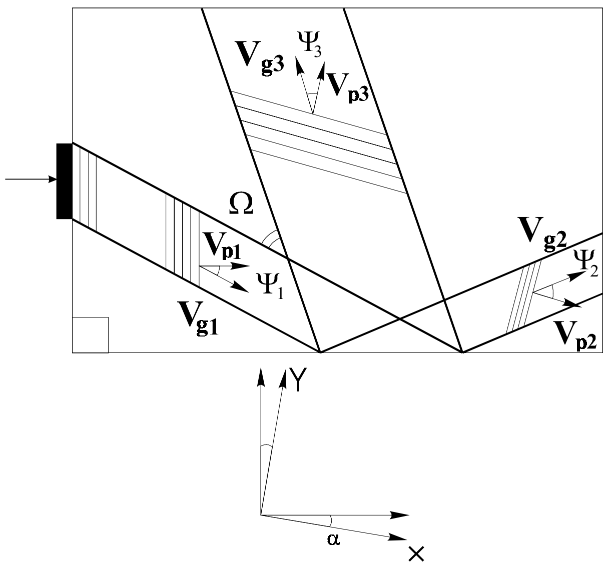

Figure 5. Propagation and reflection of acoustic waves

It occurs that, in a medium with the strong elastic anisotropy, the separation angle Ω may appear surprisingly small. For example, the analysis carried out in paratellurite for the case of the slow shear acoustic wave incident on the free boundary of the specimen revealed the following trend. It was found that the minimal value of the separation angle may be observed in the cut of the crystal with the incident acoustic waves propagating along the direction with the angle $\alpha = 12^0$ relatively to X axis. For the case of the single crystal of paratellurite, one can find the unusual reflection with the separation angle $\Omega = 6.5^0$, as illustrated in Fig. 7. A similar backward reflection in calomel may be observed along the propagation angle $\alpha = 16^0$ relatively to the axis X . It is evident that the presented consideration is valid for the rectangular configuration of the specimen, as shown in Fig. 5.

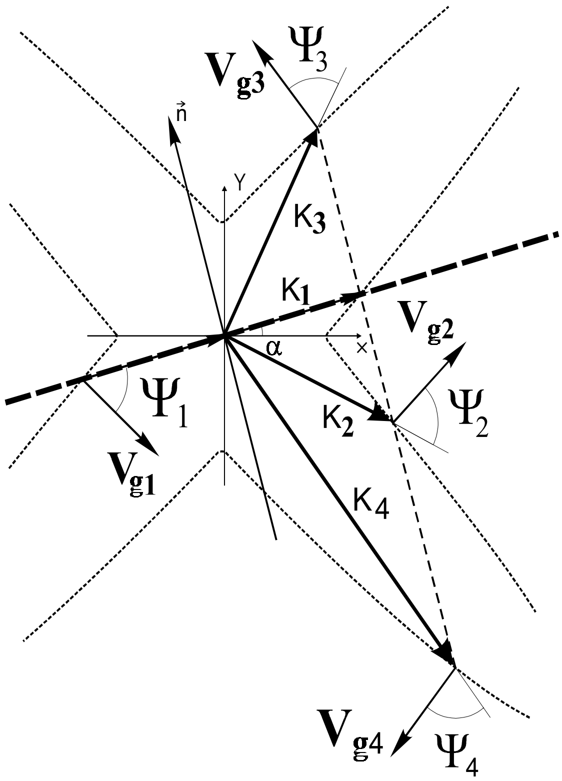

Fig. 6. Directions of acoustic phase and group velocities during the acoustic reflection

6. APPLICATION OF PECULIAR ACOUSTIC REFLECTIONS IN ACOUSTO-OPTICS

The examined peculiarities of the elastic wave propagation and reflection in the single crystals of paratellurite and calomel may be used for design of new modifications of instruments – Ref. 6 – 10. The instruments are intended for control of parameters of optical beams - Ref. 9 - 10.

For example, an efficient close to collinear tunable acousto-optic filter (AOTF) on base of the tellurium dioxide single crystal with the collinear group velocities of light and sound was proposed as a result of the present research. The filter utilized the backward propagation of the elastic energy reflected from a free optical facet in the crystal. The device possesses from 2 to 3 times lower driving acoustic power requirements in comparison with traditional close to collinear acousto-optic filters on base of TeO_2 .

The improvement in the driving acoustic and electric power is obtained in the new modification of the filter due to the conservation of the acoustic beam cross section after the reflection from the free input optical facet of the cell.

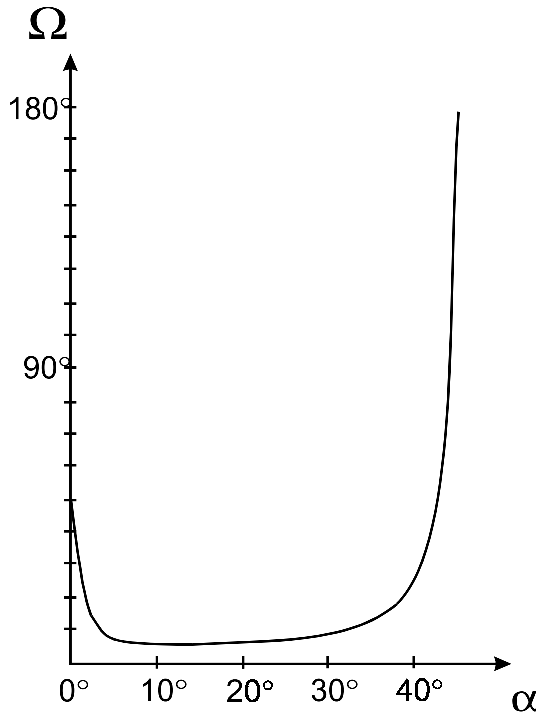

Figure 7. Dependence of spatial separation angle on cut of paratellurite single crystal

This regular trend is illustrated in Fig. 8 and Fig. 9 where the cross sections of the incident and the reflected acoustic beams are shown.

The device in Fig. 8 applies the traditional reflection of ultrasound from the free surface of the crystal. It was found that the ratio of sound phase velocities before and after the reflection in the medium appeared as great as

$$V_{p1}/V_{p3} \cong \sqrt{\frac{2\,c_{44}}{c_{11} - c_{12}}} = 3.3 \quad . \qquad (6)$$

It automatically results in the requirement on the orientation of the input optical facet relatively to the transducer facet. As found, the faced should be oriented so that the tangent projections of the incident and the reflected phase vectors of ultrasound on the boundary are equal to each other.

The carried out calculations proved that the propagation of ultrasound after the reflection along the directions close to [110] in the crystal may be provided only if the optical facet forms the angle 77^o with the bond facet of the specimen. Since the tilt angle of the optical facet is chosen far from 45^o, it means that the inevitable broadening of the acoustic column cross section takes place in the device. This broadening may be seen in Fig. 8 where the coloumn cross section after the reflection D and before the reflection d are coupled by the approximate relation $D = 3d$. In other words, it is reasonable to expect in the instrument, the loss in the acoustic power density to a factor of about 3.0 compared to the density of the acoustic beam before the reflection. The loss in the acoustic power leads to the increase of the driving power requirements.

The described disadvantage in the filter operation may be avoided if the strong elastic anisotropy of the crystal is taken into consideration during the filter design. The analysis proves that application of the crystal cut with the strong energy walkoff before the reflection results in the strong acoustic energy flow towards the optical facet of the crystal. In this

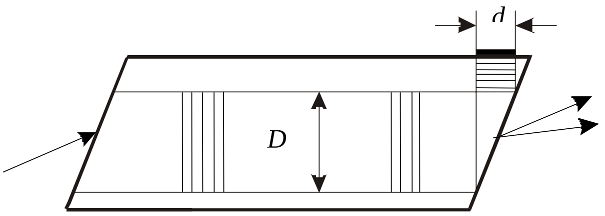

Figure 8. Configuration of acoustic column with decrease of elastic power density

case, the cross section of the acoustic column before the reflection d may be chosen approximately equal to the dimension of the column after the reflection D . Therefore, no losses in the density of the acoustic energy are observed in the cell because one ahs $D \approx d$. The reason is that the input optical facet is directed at the angle 45^{o} relatively to the transducer facet of the instrument.

It is evident that the carried out design of the filter with theconservation of the elastic energy occurred possible only due to the application of the strong acoustic energy walk-off in paratellurite. Moreover, a similar improvement in the filter operation is obtained if one chooses single crystals of calomel for the instrument design.

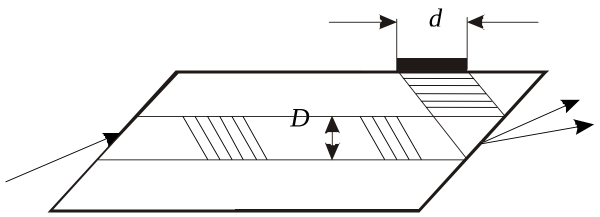

Figure 9. Configuration of the filter with conservation of elastic power density

7. CONCLUSION

Propagation and reflection of the acoustic waves in the crystals possessing the strong anisotropy of the elastic properties are characterized by many peculiarities compared to crystals without the strong elastic anisotropy. The peculiarities manifest themselves as the extraordinary strong acoustic energy walk-off as well as the unusual reflection of the elastic energy towards the incident acoustic waves.

This regular trend is not only of interest for scientists engaged in Physical Acoustics but also for the specialists dealing with the design of modern acousto-optic instruments. The tunable acousto-optic filters should be mentioned in this context.

For example, the tunable acousto-optic filter utilizing the strong elastic energy walkoff requires about 3 times lower levels of the driving electric and acoustic power compared to the corresponding instrument on base of the traditional reflection of the acoustic energy.

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